Das aktuelle DAAD-Projekt beschäftigt sich damit, wie wir Zukunftsvorhersagen treffen können, um so auch auf den Wandel in unserer Welt reagieren zu können.

Zur Einstimmung auf das bald beginnende DAAD Projekt "Modeling Change" können Sie hier den Abstract von Dr. Mario Annunziato mit dem Titel "Best Control of Uncertainty" lesen, den dieser im vergangenen Jahr im Rahmen unseres DAAD Projektes Euro-IBSA geschrieben hatte.

In our daily life we frequently face the task to maximize a profit or minimize a cost. For instance, if we want go to a place, we may ask ourselves:
Is it better use our car or the public transport? Or when purchasing clothes we might evaluate colors, costs, the quality of the fabric, that best fits our fancy and wallet. In some sense we have a "System", that is a set of possibilities and rules (paths of travel, colors availability) that defines the constraints
of our problem, and an "Objective" (time travel, costs, satisfaction) to rate the various states of the System.

A classical physical problem is to find the shape of a slide such that a sliding mass constrained on its surface, and subject to the action of the gravity force only, takes the shortest time. The problem is just to minimize a time. The solution, requiring the infinitesimal calculus, was found by Bernoulli, Newton, Leibniz, et al. at the end of the XVII Century, it is not a straight line, but the Brachistochrone curve. By summarizing, we can state that the Optimization is the process to get the best rating, but complying the constraints of the System.

Further, we can have the availability of a "Control", i.e. a tool capable of affecting the system. For example, in the problem of adjusting the temperature of a room, the thermostatic valve that is just the tool that acting on the heater allows us to perform such control. But heating the room costs money, so that we could face the problem to find a strategy in order to minimize the costs.

Moreover, we can take account of the change of the outdoor temperature during the day, and find a strategy to maximize our pleasure, and so on to extend the problem. However, the three basic ingredients of the optimal control problem are: the System, the Control, and the Objective.

The optimal control theory is a well defined branch of the mathematics. It is an hard mathematical subject that needs complex computational techniques. Calculus of variation, Lev Pontryagin, Richard Bellman, are common names related to the theory. There is an huge amount of applications of this
theory to real world problems, we quote: cost/profit optimization, shape optimization, tracking of trajectories, fast stabilization of systems.

The first important modern real time application of the optimal control theory was to the shipping a satellite into orbit. For this problem the gravity interaction between the masses of the spacecraft and the Earth, and the mass decrease of the spacecraft due to fuel consumption, have to be take into account in order to minimize the consumption, while at the same time the trajectory must be controlled. To solve this task we need a real time computation that is beyond the human capability,
so that numerical methods for automatic computing have been developed.

The problem can be further extended by including some uncertainty on the knowledge of the system. This is typical in financial problems of business strategies, where the prices of assets and goods are subject to random fluctuations in time. As a consequence of the randomness our ability to build a control strategy of the system is reduced.

This class of problems is named "Optimal control of stochastic processes" and is a modern and difficult mathematical subject of research. The aim to control randomness sounds such as an unlikely task, indeed it is possible to give it a meaning.

In fact, as it is common to consider the average of a set of values in order to get a collective rate of it, the same is generally used in scientific literature when defining the Objective for stochastic optimal control problems. Anyway, the complete description of a random variable is given by its statistical distribution. From this fact it follows the hint of using the statistic of the randomness for the description of the System and the definition of the Objective, with the aim to control the entire shape
of a statistical distribution rather than only the average value.

This framework for the optimal control theory of stochastic processes results to be a non linear mathematical problem, whose solution can be found by requiring advanced mathematical tools such us 'partial differential equation', 'functional analysis', and numerical algorithms that are ongoing of research. Currently, these techniques have been proved to be effective on the optimal control of some models from biology, classical and quantum physics and finance.

Some technical bibliography:

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