Partial least squares path modeling using ordinal categorical indicators

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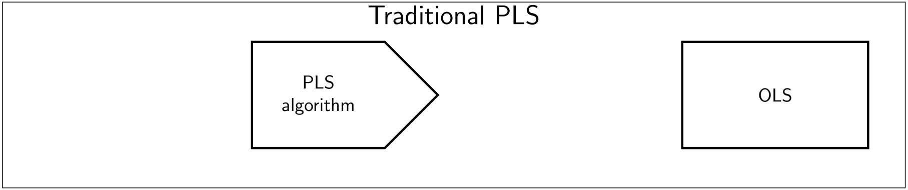
Overview

1. Partial least squares (PLS)
2. Consistent partial least squares (PLSc)
3. Literature review
4. Ordinal partial least squares (OrdPLS)
5. Consistent ordinal partial least squares (OrdPLSc)
6. Monte Carlo simulation
7. Results

Partial least squares path modeling

Traditional partial least squares [Lohmöller, 2013]

- is a variance-based estimator for SEM,
- creates composites as proxies for the theoretical constructs, and
- can be expressed in terms of indicators correlation matrices.



PLS estimation procedure

- All indicators $\boldsymbol{x}$ are standardized.
- Indicators which belong to one common factor or one composite η_j are grouped to form block j with $j = 1, ..., J$.
- The empirical correlation matrix $\boldsymbol{S}_{jj}$ of dimension $(K_j \times K_j)$ contains the correlations between the indicators of block j.

PLS estimation procedure

Traditional PLS estimation procedure consists of 3 parts.

1. Initial arbitrary outer weights $\hat{\boldsymbol{w}}_j^{(0)}$ of dimension $(K_j \times 1)$ are chosen for each block j, where $\hat{\boldsymbol{w}}_j^{(0)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(0)} = 1$.
2. Iterative PLS algorithm starts to obtain the stable final outer weights $\hat{\boldsymbol{w}}_j$ with $j = 1, ..., J$.
3. The stable weights are used to built final composites stand-ins for the constructs, and the parameters of the measurement model and the structural model are estimated by OLS.

The iterative algorithm consists of four steps:

1. Outer estimation of η_j: $\hat{\boldsymbol{\eta}}_j^{(i)} = \boldsymbol{X}_j \hat{\boldsymbol{w}}_j^{(i)}$ with $\hat{\boldsymbol{w}}_j^{(i)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(i)} = 1$
2. Inner estimation of η_j: $\tilde{\boldsymbol{\eta}}_j^{(i)} = \sum_{j'=1}^{J} e_{jj'}^{(i)} \hat{\boldsymbol{\eta}}_{j'}^{(i)}$, where

$$e_{jj'}^{(i)} = \begin{cases} \text{sign}(\hat{\boldsymbol{w}}_j^{(i)\prime} \boldsymbol{S}_{jj'} \hat{\boldsymbol{w}}_{j'}^{(i)}), \text{ for } j \neq j' & \text{if construct } j \text{ and } j' \text{ are adjacent} \\ 0, & \text{otherwise,} \end{cases}$$

Again, inner estimates are scaled to variance of one.

(i) : iteration counter

3. New outer weights are calculated:
   - Mode A (correlation weights):
     $$\hat{\boldsymbol{w}}_j^{(i+1)} \propto \sum_{j'=1}^{J} \boldsymbol{S}_{jj'} \hat{\boldsymbol{w}}_{j'}^{(i)} e_{jj'}^{(i)} \quad \text{with} \quad \hat{\boldsymbol{w}}_j^{(i+1)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(i+1)} = 1.$$
   - Mode B (regression weights):
     $$\hat{\boldsymbol{w}}_j^{(i+1)} \propto \boldsymbol{S}_{jj}^{-1} \sum_{j'=1}^{J} \boldsymbol{S}_{jj'} \hat{\boldsymbol{w}}_{j'}^{(i)} e_{jj'}^{(i)} \quad \text{with} \quad \hat{\boldsymbol{w}}_j^{(i+1)\prime} \boldsymbol{S}_{jj} \hat{\boldsymbol{w}}_j^{(i+1)} = 1.$$
4. Check for convergence: final weights $\hat{\boldsymbol{w}}_j$ are obtained if weights do not significantly change.

(i) : iteration counter

3. part: estimation of the model parameters

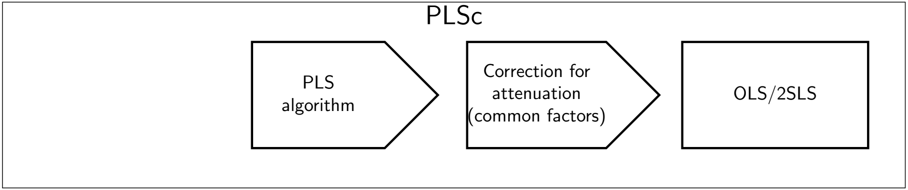
In the last part, final composites are built, $\hat{\boldsymbol{\eta}}_j = \boldsymbol{X}\hat{\boldsymbol{w}}_j$, and the model parameters are estimated:

- Parameters of the measurement model:
  - Composites: estimated weights equal the final weights
  - Common factors: factor loadings are estimated by OLS in accordance to the measurement model.
- Parameters of the structural model are estimated by OLS.

Overview

Consistent partial least squares (PLSc)

PLS estimates are biased in the case of constructs modeled as common factors.
$\rightarrow$ Consistent partial least squares produce consistent estimates for common factor models using a correction factor
[Dijkstra, T.K. & Henseler, J., 2015]



Consistent partial least squares (PLSc)

The correction factor can be calculated as follows:

$$\hat{c}_j^2 = \frac{\hat{\boldsymbol{w}}_j'(\boldsymbol{S}_{jj} - \text{diag}(\boldsymbol{S}_{jj}))\hat{\boldsymbol{w}}_j}{\hat{\boldsymbol{w}}_j'(\hat{\boldsymbol{w}}_j\hat{\boldsymbol{w}}_j' - \text{diag}(\hat{\boldsymbol{w}}_j\hat{\boldsymbol{w}}_j'))\hat{\boldsymbol{w}}_j}.$$

Consistent factor loading estimates:

$$\hat{\boldsymbol{\lambda}}_j = \hat{c}_j\hat{\boldsymbol{w}}_j$$

Consistent correlation estimates between common factors:

$$\widehat{\text{cor}(\eta_j, \eta_{j'})} = \frac{\hat{\boldsymbol{w}}_j'\boldsymbol{S}_{jj'}\hat{\boldsymbol{w}}_{j'}}{\sqrt{\hat{c}_j^2\hat{\boldsymbol{w}}_j'\hat{\boldsymbol{w}}_j\hat{c}_{j'}^2\hat{\boldsymbol{w}}_{j'}'\hat{\boldsymbol{w}}_{j'}}}$$

Overview

Suggestions provided by the literature

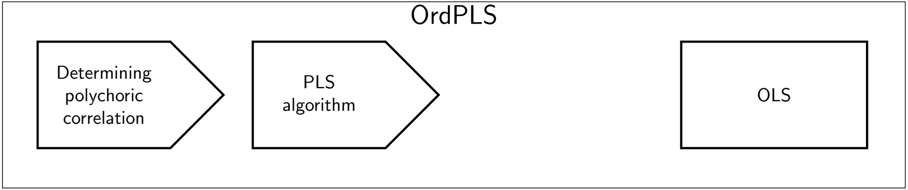
- Replace ordinal categorical indicator by a dummy matrix
- Use correspondence analysis to quantify the ordinal categorical indicator [Betzin, J. & Henseler, J., 2005]
- Partial maximum likelihood partial least squares [Jakobowicz, E.& Derquenne, C., 2007]
- Non-metric partial least squares [Russolillo, G., 2012]
- Ordinal partial least squares [Boari, G., & Cantaluppi, G., 2012]

To our knowledge, no approaches for PLSc dealing with ordinal categorical indicators.

Overview

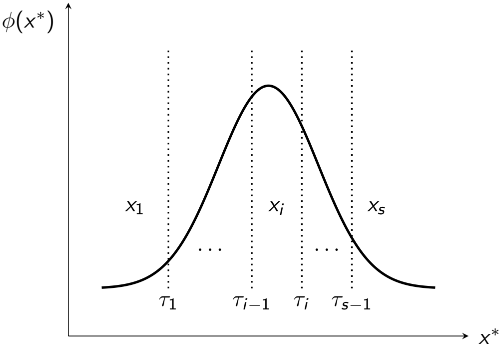
Ordinal partial least squares (OrdPLS)

Ordinal partial least squares (OrdPLS) [Boari, G., & Cantaluppi, G., 2012] uses the polychoric correlation as input for the PLS algorithm instead of the product-moment correlation.



Polychoric correlation

Assumption: An ordinal categorical indicator x is the result of a polytomized standard normally distributed random variable x^*



$$x = x_i \quad \text{if} \quad \tau_{i-1} \leq x^* < \tau_i \quad i = 1, ..., s$$

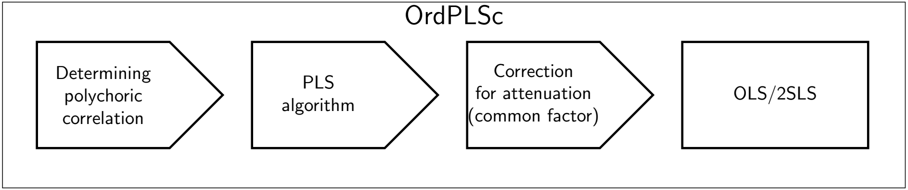
Polychoric correlation [Olsson, U., 1979, Poon, W.-Y. & Lee, S.-Y., 1987]: estimated correlation between the underlying latent variables

Overview

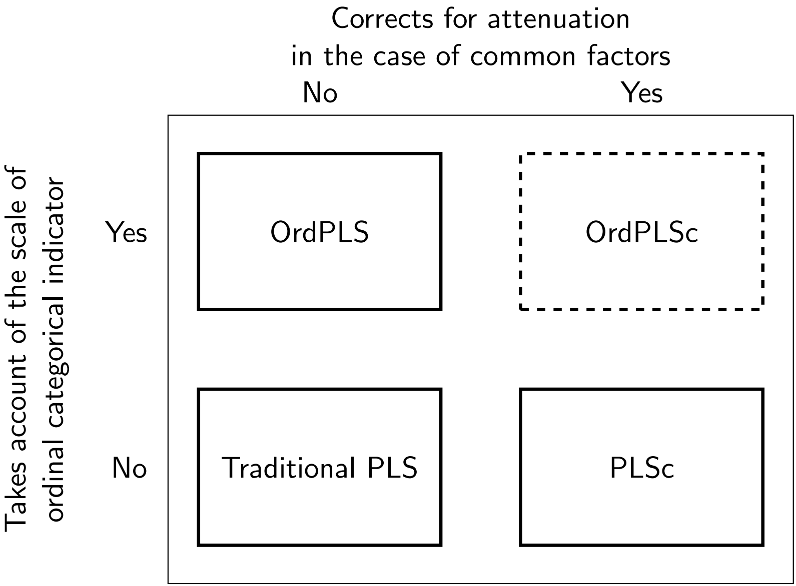
Consistent ordinal partial least squares (OrdPLSc)

OrdPLSc combines the idea of OrdPLS and PLSc

- Corrects for attenuation if common factors are included in the model
- Uses the polychoric correlation as input for the PLS algorithm



Ordinal consistent partial least squares (OrdPLSc)



Overview

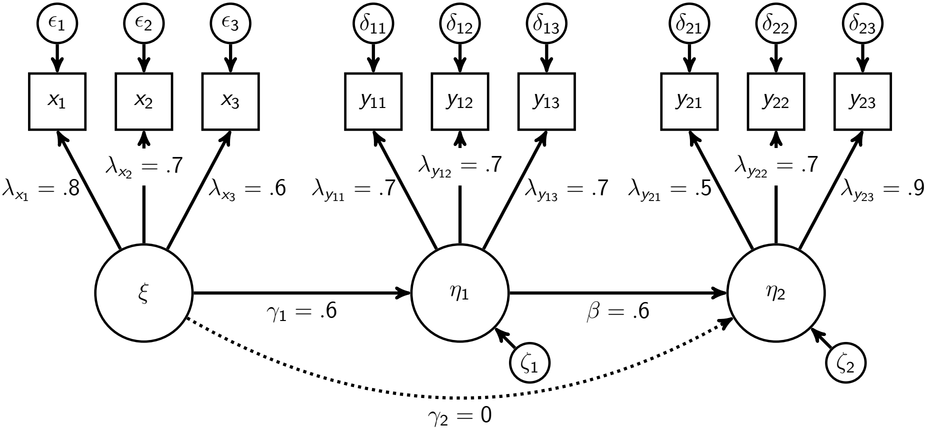
Monte Carlo simulation

We considered a population model with three common factors and varied:

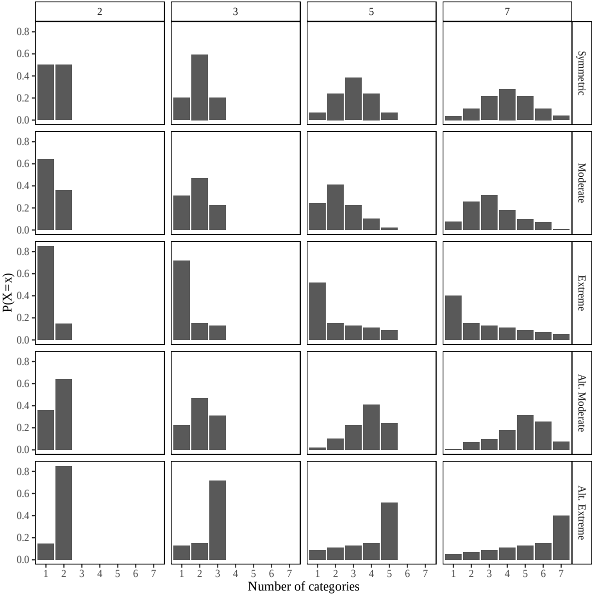
- Number of categories: 2, 3, 5, and 7 categories
- Skewness of the indicators

We create 1000 data sets ($N = 500$) for each design.

Population model with three common factors



Skewness of indicators



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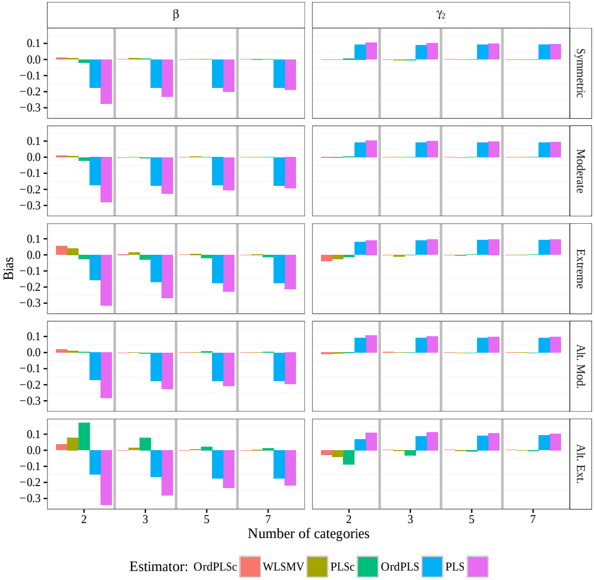
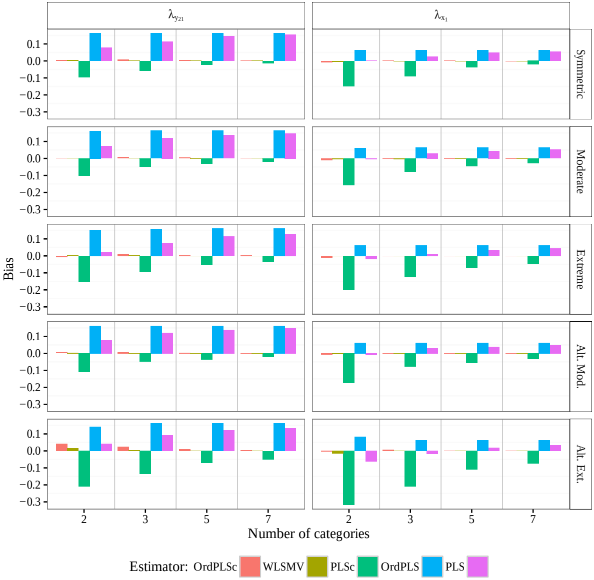
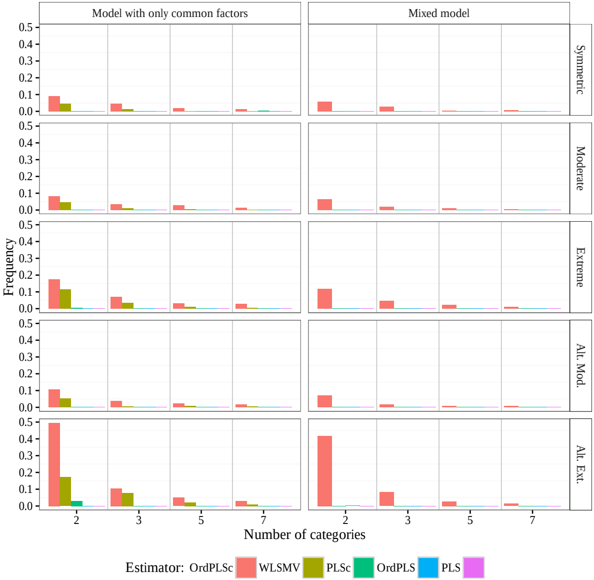
Results: bias of the path coefficient estimates



Results: bias of the factor loading estimates



Results: inadmissible results



Summary

- OrdPLSc and WLSMV led to almost the same results.
- PLSc path coefficient estimates behaved surprisingly well, while factor loadings estimates were biased.
- OrdPLS estimates were fairly constantly biased .
- The bias of PLS estimates converged to the bias of OrdPLS estimates with an increasing number of categories.

Thank you!
Questions/Comments?

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